

Name:

Date:

Period: 2

Perpendicular and Angle Bisectors

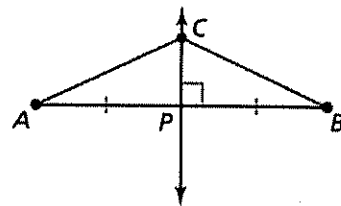
Lesson Objective INBAT use $\perp$ and $\angle$ bisectors to find measures and distances; write eqns of $\perp$ bisectors

In Chapter 3, you learned that a *perpendicular bisector* of a line segment is the line that is $\perp$ to the segment at its midpoint.

Equidistant: A point is equidistant from two figures when the point is the same distance from each figure.

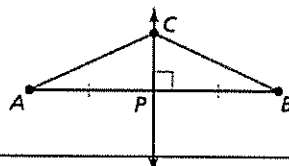
Perpendicular Bisector Theorem

In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.



PROOF: Given: $\overline{CP}$ is the $\perp$ bisector of $\overline{AB}$

Prove: $CA = CB$



Statements

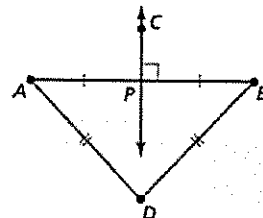
Reasons

- ① $\overline{CP}$ is the $\perp$ bisector of $\overline{AB}$
- ② $\overline{AP} \cong \overline{BP}$
- ③ $\overline{CP} \cong \overline{CP}$
- ④ $\angle CPA$ and $\angle CPB$ are rt. $\angle$ s
- ⑤ $\angle CPA \cong \angle CPB$
- ⑥ $\triangle CPA \cong \triangle CPB$
- ⑦ $\overline{CA} \cong \overline{CB}$
- ⑧ $CA = CB$

- ① given
- ② def. of a seg. bisector
- ③ reflex. prop. $\cong$
- ④ $\perp \rightarrow$ rt. $\angle$ s
- ⑤ all rt. $\angle$ s are $\cong$
- ⑥ SAS $\cong$ (2, 5, 3)
- ⑦ CPCTC
- ⑧ $\cong$ segs. are =

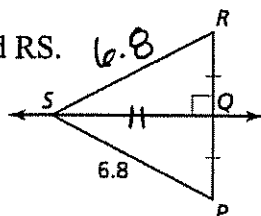
Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the $\perp$ bisector of the segment.



Examples:

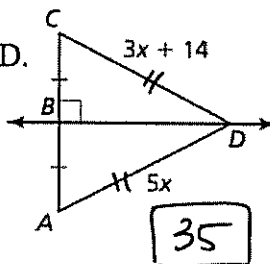
1. Find RS.



2. Find AD.

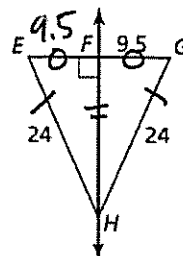
$$3x + 14 = 5x$$

$$x = 7$$



3. Find EG.

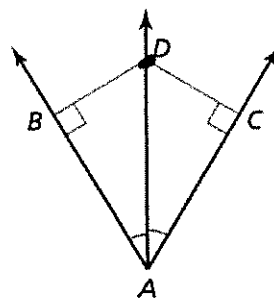
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In Chapter 1, you learned that an *angle bisector* is a ray that divides an angle into two congruent adjacent angles.

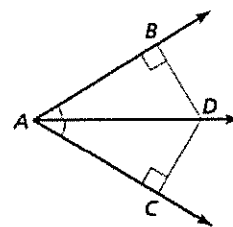
You also learned that the *distance from a point to a line* is the length of the $\perp$ segment from the point to the line.

So, in the figure at right, $\overrightarrow{AD}$ is the bisector of $\angle BAC$, and the distance from point D to $\overrightarrow{AB}$ is DB, where $\overline{DB} \perp \overrightarrow{AB}$

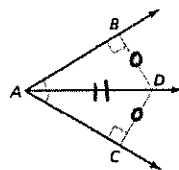


Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.



PROOF: **Given:** $\overrightarrow{AD}$ bisects $\angle BAC$, $\overline{DB} \perp \overrightarrow{AB}$, $\overline{DC} \perp \overrightarrow{AC}$
Prove: $DB = DC$



Statements

Reasons

① $\overrightarrow{AD}$ bisects $\angle BAC$
 $\overline{DB} \perp \overrightarrow{AB}$, $\overline{DC} \perp \overrightarrow{AC}$

① given

② $\angle BAD \cong \angle CAD$

② def. of $\angle$ bisector

③ $\angle DBA$ and $\angle DCA$ are rt. $\angle$ s

③ $\perp \rightarrow$ rt. $\angle$ s

④ $\angle DBA \cong \angle DCA$

④ all rt. $\angle$ s are $\cong$

⑤ $\overline{AD} \cong \overline{AD}$

⑤ reflex. prop. $\cong$

⑥ $\triangle DBA \cong \triangle DCA$

⑥ AAS $\cong$ (2, 4, 5)

⑦ $\overline{DB} \cong \overline{DC}$

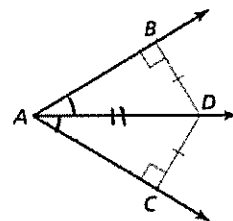
⑦ CPCTC

⑧ $DB = DC$

⑧ $\cong$ segs. are =

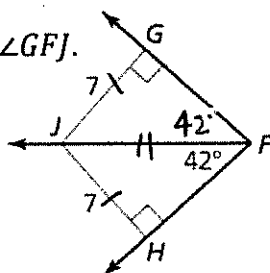
Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.

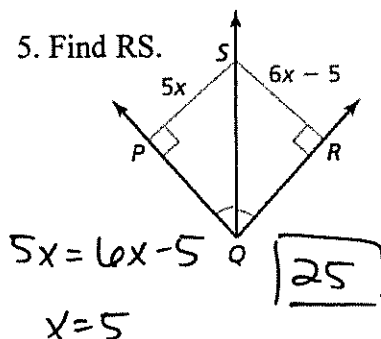


Examples:

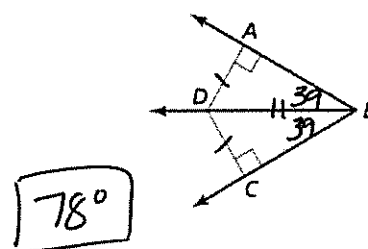
4. Find $m\angle GFJ$.



5. Find RS.



6. Find $m\angle ABC$ when $AD = 3.2$, $CD = 3.2$ and $m\angle DBC = 39^\circ$



Writing Equations of Perpendicular Bisectors

Example:

7. Write an equation of the perpendicular bisector of the segment with endpoints P(-2, 3) and Q(4, 1)

Step 1: Find the midpoint of the original segment. Why? bisector cuts seg. @ mdpt

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \rightarrow \left(\frac{-2 + 4}{2}, \frac{3 + 1}{2} \right) = \left(\frac{2}{2}, \frac{4}{2} \right) = (1, 2)$$

Step 2: Find the slope of the original segment. Why? need it to get $\perp$ slope

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{4 - (-2)} = \frac{-2}{6} = -\frac{1}{3}$$

Step 3: Find the slope of the segment *perpendicular* to the original segment. How? product of slopes is -1

$$\left(-\frac{1}{3}\right)(m) = -1$$
$$\boxed{m = 3}$$

Step 4: Write the equation.

Use point-slope form ($y - y_1 = m(x - x_1)$) OR slope-intercept form ($y = mx + b$)

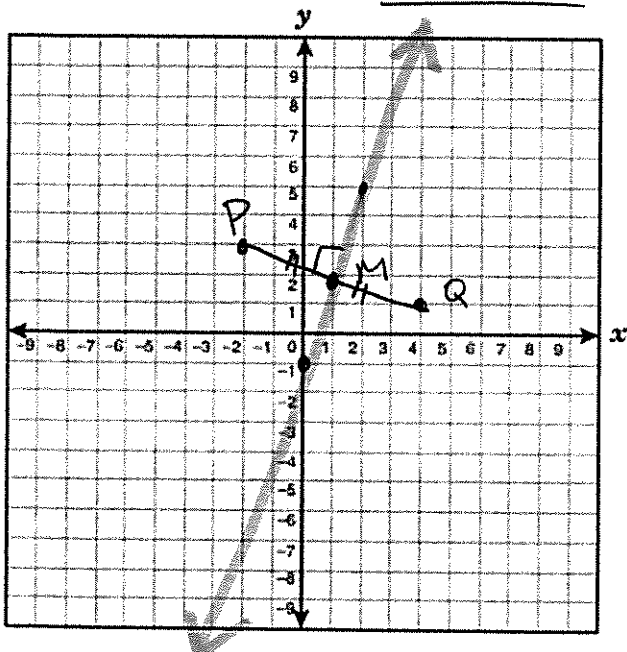
$$\downarrow$$
$$y - 2 = 3(x - 1)$$
$$y - 2 = 3x - 3$$
$$\boxed{y = 3x - 1}$$

$$2 = 3(1) + b$$

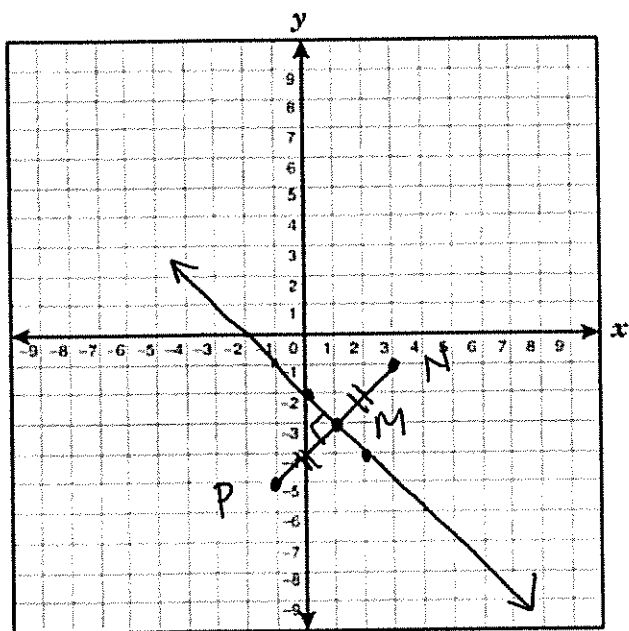
$$2 = 3 + b$$

$$-1 = b$$

$$\boxed{y = 3x - 1}$$



8. Write the equation of the perpendicular bisector of the segment with endpoints P(-1, -5) and N(3, -1)



$$\text{mdpt} = \left(\frac{-1+3}{2}, \frac{-5+(-1)}{2} \right)$$

$$= \left(\frac{2}{2}, -\frac{6}{2} \right)$$

$$= (1, -3)$$

$$\text{slope}^{\text{orig.}} = \frac{-1 - (-5)}{3 - (-1)}$$

$$= \frac{4}{4}$$

$$= 1$$

$$\perp \text{ slope} = -1$$

$$y - (-3) = -1(x - 1)$$

$$y + 3 = -x + 1$$

$$\boxed{y = -x - 2}$$

Not nice #s example

Find the eqn of the $\perp$ bisector
w/ endpoints $Y(10, -7)$ and $Z(-4, 1)$

$$\begin{aligned}\text{mdpt} &= \left(\frac{10 + (-4)}{2}, \frac{-7 + 1}{2} \right) \\ &= \left(\frac{6}{2}, -\frac{6}{2} \right) \\ &= (3, -3)\end{aligned}$$

$$\text{orig. slope} = \frac{-7 - 1}{10 - (-4)} = \frac{-8}{14} = -\frac{4}{7}$$

$$\perp \text{ slope} = \frac{7}{4}$$

$$y - (-3) = \frac{7}{4}(x - 3)$$

$$y + 3 = \frac{7}{4}x - \frac{21}{4}$$

$$y = \frac{7}{4}x - \frac{21}{4} - 3$$

$$y = \frac{7}{4}x - \frac{21}{4} - \frac{12}{4}$$

$$\boxed{y = \frac{7}{4}x - \frac{33}{4}}$$

Name: _____

Find each solution set by factoring.

Example: $x^2 - x - 6 = 0$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - 15 = 0$

$$0 = 8 + x9 + x^2 \boxed{}$$

$$x^2 - 7x + 12 = 0$$

$$0 = 3 - x + 10x^2 \quad \square$$

$$8x^2 - 2x - 3 = 0$$

$$2x^2 + 13x + 15 = 0$$

$$0 = 91 + 8x - x^2$$

$$x_2 - 10x = 0$$

$$2x^2 + x - 6 = 0$$

$$3x^2 - 14x - 5 = 0$$

$$0 = 6 - x$$

$$x^2 - 4x + 4 = 0$$

$$0 = 9 - 13x + 6x^2$$

$$15x^2 - 17x + 4 = 0$$

[illegible]